An Experimental Investigation of Flow Mode Selection in a
Conical Taylor-Couette System

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Transitions of viscous flow between coaxial conical cylinders with the inner one rotating
and the outer one at rest were investigated to reveal mode selection of the first instabilities
with the aid of flow visualization and spectral analysis. The rotational velocity of the
inner conical cylinder was linearly accelerated from rest until reaching its final speed.
The different observed states were successfully distinguished by their dependency on the
acceleration rate $b$ in the investigated range of Taylor number $Ta$. Transitions between
states were determined as functions of $\beta$ and $Ta$ by fixing the Taylor number and varying
the acceleration rate in the range $0.01-1.5$ rad/s$^2$. Observed states were classified into:
first toroidal vortices (FTV), helical motion (HMV), upward travelling vortices (UTV),
steady Taylor vortices (TVF) and wavy vortices (WVF). Modes of six pairs of Taylor
vortices (6TVF), seven pairs (7TVF) and eight pairs (8TVF) were observed at the same
$Ta$ and different $\beta$. Steady Taylor vortices and wavy vortices were also observed when $\beta$
increased at the same $Ta$. The spectral analysis indicated that the states HMV and WVF
have constant ratios between the characteristic frequencies and the frequency of rotation
of the inner conical cylinder, while in UTV the ratio decreases with increasing $Ta$. The
mode selection diagram in the ($Ta$, $\beta$) plane has no regular form with regard to the zones
delimiting the different observed states.

1. INTRODUCTION

The flow between coaxial rotating cylinders has been investigated by many researchers
as a system suitable for the study of transition to turbulence. Its geometric symmetry has
made the study of the laminar regimes highly successful compared to more general flows.
It has been found that the number of flow states involved in the transition region until the
appearance of turbulence does not need to be large. A succession of only a finite number of
transitions leads this flow system to the onset of turbulence (Andereck et al. 1986). However,
the onset of the first instability controls the path followed by the transition from one mode
to another. Many authors have reported the non-uniqueness of the observed states. Coles (1965) counted up to 25 different modes observed, which depend not only on the initial conditions but also on the manner in which the inner cylinder was accelerated to the final speed. Following the work of Coles (1965), some other authors investigated the influence of the acceleration on the establishment of the final state (Andereck et al. 1986, Burkhalter & Koschmieder 1974). Recently, the effect of the acceleration was characterized by controlling quantitatively the acceleration rate involved. Kataoka (1998) dealt with the case of linear acceleration of the inner cylinder and discussed its consequence on the observed modes. Lim et al. (1998) also applied linear accelerations to investigate Taylor vortex flow (TVF) and wavy vortex flow (WVF). They found a new flow regime with a resemblance to regular TVF, the flow pattern of which presents a shorter wavelength. In other rotating systems with constant gap widths, Taylor vortices have also been observed as in the flow between coaxial spheres (Wimmer 1976) and the flow between coaxial conical cylinders (Wimmer 1995, Noui-Mehidi & Wimmer 1999). In the latter flow system, the basic flow is three-dimensional and the flow transitions are highly sensitive to initial conditions. Owing to the conical radii, the centrifugal force changes axially and increases with increasing radii. The hydrodynamics of rotating or swirling motions are geometrically affected in conical systems. Noui-Mehidi et al. (1999) found that the swirl properties in a conical system compared to those in a cylindrical geometry can enhance the overall mass transfer by 15%. The experiment of Wimmer (1995) showed the non-uniqueness of the TVF flow by accelerating the inner conical cylinder with different rates. Despite the originality of the work of Wimmer (1995), however, quantitative estimation of the acceleration rates used was not discussed. The present investigation was carried out with the aim of providing a clear quantitative understanding concerning the effect of the acceleration rate on the onset of the different flow modes observed between the conical cylinders.

2. EXPERIMENTAL CONDITIONS

2.1. THE CONCENTRIC CONICAL CYLINDER SYSTEM

The experimental apparatus in Figure 1 consists of an inner conical cylinder of stainless steel and an outer transparent conical cylinder of acrylic resin. The inner one is rotated and the outer one remains at rest. The maximum radii of the inner and outer conical cylinders are \( R_{ih} = 42 \text{ mm} \) and \( R_{oh} = 50 \text{ mm} \), respectively, at the top of the flow system, where the radius ratio is \( h = 0.84 \). The two conical bodies have the same apex angle \( \alpha = 16 \text{ degrees} \), which makes the gap width constant and equal to \( d = 8 \text{ mm} \). The height of the working fluid column \( L = 125 \text{ mm} \) gives the aspect ratio \( G = 15.62 \). The end plates are fixed to the outer conical cylinder. The outside of the outer conical cylinder is rectangular for flow visualization. The working fluid was an aqueous solution of 66 vol% glycerol, and 2% of Kalliroscope AQ1000 was added for flow visualization. The temperature was measured by a thermo-couple of Copper/Constantan with an accuracy of 0.1 C . The kinematic viscosity of the solution was 14.82 cm\(^2\)s\(^{-1}\) at 25 C. The flow structure visualized with kalliroscope was observed by the use of a high resolution video camera. Two illumination techniques were utilized: Argon laser sheet illumination for observing a cross section of the gap between the conical cylinders, and reflected white light illumination for observing the front view of the flow system.

In the present experiment, time-dependent flow states were studied by analyzing the
intensity of a He-Ne laser light scattered by the Kalliroscope flakes. The signals are collected and filtered by the use of DSP software via a personal computer. Frequency spectra are then obtained from time-series records of the laser light intensity signals.

Since the conical radii vary axially, it is rather difficult to define a representative constant Reynolds number as distinct from that for a circular cylindrical system. In the case of concentric rotating cylinders, the Reynolds number is defined as:

$$Re = \frac{R_{th} \Omega \rho d}{\nu},$$

where $\Omega$ is the angular velocity and $\nu$ is the kinematic viscosity. Previous works dealing with conical geometries, deduced a constant Taylor number defined by:

$$Ta = \frac{d^2 \Omega \sin \alpha}{\nu}$$

from theoretical considerations after reducing the equations of motion written in a curvi-linear coordinate system (Bark et al. 1984, Noui-Mehidi & Bouabdallah 1993). This Taylor number can be written as:

$$Ta = \frac{d}{R_{th}} \sin \alpha Re = \eta \sin \alpha Re,$$
where \( \eta \) is the radius ratio (Cognet 1984). Thus, in the definition of the Taylor number, the conicity of the geometry is expressed in terms of the classical Reynolds number which is not constant.

2.2. ACCELERATION CONTROL

The inner conical cylinder was rotated by a DC motor which can give a stable speed in the range \( 0 \sim 5000 \) rpm. A speed reducer of a ratio \( 1/3 \) covered the Taylor number range \( 0 \sim 30 \) in the actual conditions. The DC motor was controlled by a PC with the use of a BASIC algorithm which permits the fixing of the acceleration path for start-up operation. The acceleration time required between the initial and final speed and the final voltage set in the algorithm can give an arbitrarily desired acceleration rate. In the present work, linear accelerations are investigated. The inner conical cylinder is accelerated from rest until its final speed according to the relation:

\[
\Omega(t) = \beta t,
\]

where \( \beta \) is the acceleration rate and \( t \) is the time. The acceleration rate can be changed in the range between \( 0.01 \) and \( 1.5 \) rad/s\(^2\). In order to assure steadiness of the flow states, the observations commenced after a duration of steady rotation equivalent to 100 times the acceleration time from rest to the final speed.

3. RESULTS AND DISCUSSION

3.1. PRINCIPAL RESULTS

The different flow states observed are presented in the diagram shown in Figure 2. It has been found that this diagram comprises many zones corresponding to different flow modes demarcated in the \((Ta, \beta)\) plane. The labels of the flow modes observed are summarized in Table 1.

The basic flow in this system is three-dimensional and depends on the angular velocity \( \Omega \) of the inner conical cylinder, the gap width \( d \), the apex angle \( \alpha \) and also the axial position as reported by Wimmer (1995). Noui-Mehidi & Bouabdallah (1993) showed in previous work that for laminar flow, the mean wall velocity gradients on the inside wall of the outer conical cylinder decrease with decreasing radii in a concentric rotating conical system by the use of an electrochemical method. When the inner conical cylinder is linearly accelerated from rest, the first transition in the present system occurs at the critical value \( Ta_c = 3.49 \). This value represents the first transition boundary common for all the investigated acceleration rates. The first instability occurs due to the birth of a single vortex rotating inward along the upper end plate. In the remaining part of the fluid column, the fluid motion is regulated by the meridional circulation which is directed upward along the rotating wall and downward along the fixed one. When the speed is slowly increased above \( Ta_c \), more vortices are generated, one below the other from the top, as represented by the photographs in Figure 3. This state is labeled FTV. The neighboring vortices are counter-rotating. When three quarters of the fluid column is filled by the vortices, two branches can be obtained depending upon the acceleration rate as schematically summarized in Figure 4. For acceleration rates varying from \( 0.01 \) to \( 0.07 \) rad/s\(^2\), i.e. for very low acceleration rates, a helical motion takes place in
the flow system at the critical value $Ta_b = 5.11$ from the top corresponding to the branch 1 of Figure 4. At the value $Ta_b$, if the acceleration rate is between 0.07 and 1.5 rad/s$^2$, the branch 2 of Figure 4 is followed. That is, as $Ta$ increases from $Ta_b$, with a constant acceleration rate, the first visualized vortices take an upward travelling motion from the bottom to the top (the largest radius). At this state the whole fluid column is filled with vortices. This upward motion diminishes with increasing $Ta$ until it stops, and then a train of steady Taylor vortices is obtained. For higher speeds, Taylor vortices become wavy. The flow states in Figure 2, have been reported after repeating the experiments at least ten times for each measurement point, which gives an accuracy better than 2% for the drawing of the boundaries delimitating the different zones. These flow states will be discussed in detail in the following paragraphs.

### 3.2. HELICAL MOTION

As stated before, with an acceleration rate lower than 0.07 rad/s$^2$ above $Ta_b$, a helical motion is established on the branch 1 after the FTV state. This helical motion has its birth at the top of the flow system. The whole fluid column is filled with the helical vortices as shown in Animation 1, played in normal speed mode. These helical vortices are inclined to the horizontal plane with the inclination angles varying axially. The inclination angle $\gamma$ decreases with axially increasing radii (Figure 5). In this state, the cross-sectional visualization, presented in Figure 6b, shows that the vortices are alternately small and large. The larger cells are closer to the outer wall while the small ones are closer to the inner wall. In the range of the Taylor numbers studied $0 < Ta < 30$ the downward motion of the helix increases with increasing Taylor number. The power spectra obtained from time series scattered of light intensity at the midpoint of the fluid column showed that the velocity of the helix is in a constant ratio to the peripheral velocity of the inner rotating cone. In the spectrum of Figure 7, a single sharp component is present with its harmonics and indicates the periodical passage of the helical vortices by the observation point. The ratio of the frequency of the helix $f_H$ to the frequency of rotation $f_r$ is 0.38 and remains constant even when $Ta$ increases in the studied range. This value also has been obtained.
Figure 2. Mode Selection Diagram. The labels are defined in Table 1.


3.3. UPWARD TRAVELLING MOTION

In the second branch represented in Figure 4, above the critical value of $Ta_b$, begins the state of the upward travelling vortices. This flow state takes place when the top 3/4 section of the fluid column is filled with the first observed vortices (i.e., the FTV state) as reported before. The vortices, still horizontal, start to move in a global upward motion due to the strong meridional circulation present in the remaining 1/4 of the fluid column still in basic flow. According to the acceleration rate values presented in Figure 2, at the Taylor number $Ta = 5.54$, the whole fluid column is filled with horizontal vortices travelling upward since more vortices appear in the lower part. This upward motion is regulated by the periodical appearance of a new vortex at the bottom of the fluid column to compensate for the disappearance of a vortex crashing onto the uppermost vortex at the top. The uppermost vortex has a strong vorticity and is not affected by this periodical phenomenon. Since the upward motion is very slow, Animation 2 is displayed at higher than actual speed to permit easier visualization of the phenomenon. The disappearance of the moving vortex crashing on the uppermost vortex can be clearly seen in this animation. The velocity $c$ of the upward motion as observed decreases with increasing Taylor number as shown in Figure 8. A single vortex travels with a very slow constant speed in the upward motion and the velocity is simply evaluated from the time spent for this vortex to travel over a fixed axial length. The phenomenon is slow enough to make measurements with good accuracy. It is worth noting from Figure 8, that the velocity of the upward motion does not depend on the acceleration rate $\beta$. The power spectrum representing this flow state is
Figure 3. Visualisation of the first vortices (FTV) with argon laser sheet illumination. Corresponding Taylor numbers: a) $Ta = 2.8$; b) $Ta = 4.2$; c) $Ta = 4.9$.

Figure 4. Schematic representation of bifurcation branches.

shown in Figure 9. There is a sharp peak component $f_{up}$ related to the passing frequency of the upward moving vortices. During the decrease of the upward motion when the Taylor number increases, the ratio $f/f_r$ is not constant as noticed in all the obtained spectra.
3.4. THE STEADY TAYLOR VORTICES

The upward travelling motion diminishes as discussed in the previous paragraph until its stops and then a train of steady vortices is established in the flow system at a critical value
\( Ta_4 = 8.48 \) with \( \beta = 0.085 \). Even if the velocity of the upward motion does not depend on the acceleration rate \( \beta \), configuration in six pairs, seven pairs or eight pairs of steady Taylor vortices can be obtained depending upon \( \beta \) in the TVF region above \( Ta_4 \). This implies how fast the point where the upward motion stops is reached. That is, if the value
of $\beta$ is high, the upward motion stops after a short time, leading to eight pairs of vortices. For lower values of $\beta$, the time required for the decrease of the upward motion becomes longer such that finally only seven or six pairs of vortices are established. This property is supported by Figure 2 where this behavior can be deduced from the values of $\beta$ and $Ta$. In the case of circular rotating cylinders, the non-uniqueness of the flow modes has been observed (Coles 1965, and Burkhalter & Koschmieder 1974). In his early observations, Wimmer (1995) reported also these flow properties for different gaps between the cones. In the present work, a quantification of the acceleration effects on the mode selection between the cones has been achieved by the mapping presented in Figure 2. The visualization of the three observed modes, presented in Figure 10, shows that in all the three observed cases, the vortices are displayed as a pair of large and small cells. In this state, the sizes of these cells decrease when the conical radii increase: the larger vortex displayed at the bottom of the fluid column is clearly greater than the one at the top. In the whole fluid column, a larger vortex and an adjacent smaller one are counter-rotating. The classical definition of the axial wavelength $\lambda$ as for a Taylor-Couette system between circular rotating cylinders can be adopted in the present case as a pair of vortices periodically displayed as the adjacent larger and smaller counter-rotating cells. The first pair of vortices is formed by the largest vortex displayed at the bottom of the flow system and the next small one. The middle point $z_m$ at the centerline of each pair is taken as the axial location for the wavelength. In Figure 11, the wave number $\Lambda^*$ obtained by

$$\Lambda^* = \frac{2\pi d}{\lambda},$$

is displayed with regard to the axial position

$$z^* = \frac{z_m}{L}.$$

As observed, the wavenumber $\Lambda^*$ increases when the conical radii increase.
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Figure 10. Visualisation of three different modes: 6TVF, 7TVF, 8TVF. The parameters for the three cases are: a) $T_a = 9.22, \beta = 0.24$; b) $T_a = 13.69, \beta = 0.37$; c) $T_a = 17.81, \beta = 1.25$.

Figure 11. Power spectrum of 6WVF state.

The most preferred arrangement in the present flow system in the TVF region is the one of seven pairs of vortices (7TVF) as it occupies the widest part in the diagram of Figure 2. The region corresponding to the TVF in the interval $8.48 < T_a < 22.53$ is not of regular form. A combination of two flow modes can be observed for the same $T_a$ and different $\beta$. As the frontiers separating the 7TVF from the 6TVF and from the 8TVF are not linear in Figure 2, modes with six pairs and seven pairs can be observed for a fixed $T_a$ in the lower border of the 7TVF according to $T_a$. In the same way, in the region delimiting the 7TVF and the 8TVF regions, seven and eight pairs can be observed when $T_a$ is fixed and $\beta$ is varied. The 8TVF is confined to a region reached only for high values of $T_a$ and $\beta$ such as:
3.5. THE WAVY VORTICES

The wavy-vortex modes represented in Figure 2 are similar in many respects to wavy-vortex flows between circular rotating cylinders (Cognet 1984). For higher Taylor numbers, the modes of 6TVF, 7TVF and 8TVF become wavy at different critical values of $Ta$ with respect to $\beta$. For example, Animation 3 corresponds to seven pairs of wavy vortices and is played at normal speed (real time). These wavy flow modes consist of azimuthal waves propagating on Taylor vortices such that the boundaries of the vortices oscillate in the circumferential direction. The 6WVF mode appears in a very narrow region $14.58 < Ta < 19.88$ and for $0.07 < \beta < 0.09$. The 7WVF and 8WVF modes are observed in the range $17.23 < Ta < 27$. Due to the irregularity of the delimiting zones, the three modes 6WVF, 7TVF and 8TVF can be observed at the same Taylor number and different values of $\beta$ as displayed in Figure 2. On the other hand, another combination of 6WVF, 7WVF and 8TVF can also be seen for a fixed $Ta$ and different $\beta$. The power spectra corresponding to the wavy modes are presented in Figure 12, 13 and 14 respectively for 6WVF, 7WVF and 8WVF. These spectra, indicating a single wavy vortex mode, have been obtained from an observation point situated at the axial middle distance of the fluid column. In each of the wavy modes, the ratio of the frequency of the wavy pattern to the inner cone rotational frequency has been found to be constant. This ratio takes the values 0.34, 0.32 and 0.31 for 6WVF, 7WVF and 8WVF, respectively.

![Figure 12. Power spectrum of 6WVF state.](image)

$16.70 < Ta < 22.53$ and $1.16 < \beta < 1.50$. 
3.6. COMBINATIONS OF DIFFERENT MODES

In the preceding paragraphs, the acceleration rate $\beta$ was fixed following a linear acceleration of the rotation of the inner cone. In other cases, when the acceleration does not follow a linear path, combination of different modes occurs. In these modes, steady Taylor vortices and helical motion can co-exist. In the studied Taylor number range, the combi-
nation of modes represented in Animation 4 is obtained when the inner cone is accelerated very rapidly above $T_{ac}$. Animation 4 is displayed at a higher than normal speed to make the phenomenon clear. As remarked, from the top of the flow system, five steady Taylor cells are present while the sixth one below them splits periodically into two cells due to the downward helical motion present in the remaining lower part of the fluid column. While a new helical vortex is born from the splitting of the sixth vortex from the top, the lowest helical vortex crashes into the bottom end plate.

4. CONCLUSION

The control of the acceleration of the inner cone by computer control has permitted the construction of a mode selection diagram where all the observed modes are classified according to the Taylor number and the acceleration rate for $d = 8 \text{ mm}$ and a radius ratio of 0.84. Each of the flow states has been discussed according to the visualization and the spectral behavior when the structure is time-dependent. As a main result, the branching which occurs at the transition to either HMV or UTV depends strongly on the acceleration rate $\beta$. In the present system, the steady Taylor vortices (TVF) are only obtained on the second branch after a transitory upward travelling motion which leads to the establishment of three steady Taylor vortex modes (6TVF, 7TVF and 8TVF). On the first branch obtained for very low acceleration rates, a helical motion dominates the flow for the range of Taylor numbers studied. The computer animations have permitted clear visualization of the different time-dependent observed flows. When the inner cone rotation is not accelerated linearly, modes consisting of combinations of Taylor vortices and helical motion have been observed. For a linear acceleration of the rotation of the inner cone, the flow states are governed by the given mode selection diagram.

REFERENCES


Stills from Animations

The first frames from the animations available through the WWW. Top to bottom correspond to animations 1 through 4 respectively.